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MATHEMATICAL MODEL OF CATTLE DEMOGRAPHICS ON A FARM IN THE EL OCHO LETRAS REGION

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ABSTRACT

In the El Ocho Letras region, cattle ranching causes significant damage to vegetation, natural wetlands, and water resources. Thus, it is necessary to know and understand the transient behavior of the number of cattle of different age groups over time and the effect of exogenous variables on the system's behavior. To this end, system dynamics is used to describe the behavior of the quantity of livestock on the Laguna Negra property, located in the El Ocho Letras region. The numbers of cattle of different ages during the life cycle were defined as variables. The animals pass through the different ages, each lasting one year. Time delays were used for the fluxes between the different ages. A control strategy was used for limiting the number of animals older than one year. A discrete time mathematical model was formulated and a simulation was performed, showing as a result a periodical oscillatory behavior for the evolution of the variables. Some of the main contributions of this study are related to: analyzing the effect of exogenous variables on the behavior of the level variables; proposing a control of the number of cattle older than one year based on the sale of female calves; and developing a discrete time mathematical model using the chain method with delayed fluxes.

KEYWORDS: system dynamics, modeling, delayed fluxes, cattle.

MODELO MATEMÁTICO DE LA DEMOGRAFÍA DEL GANADO DE UN PREDIO DEL SECTOR EL OCHO LETRAS

RESUMEN

En el sector El Ocho Letras la ganadería es un significativo causante de la degradación de la vegetación, humedales naturales, y el recurso hídrico. Esto conlleva a la necesidad de conocer y comprender la evolución de la cantidad de ejemplares de las diferentes edades del ganado, a lo largo del tiempo, y el efecto de las distintas variables exógenas sobre el comportamiento del sistema. A este propósito, en el presente estudio se aplica la Dinámica de Sistemas para

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la representación del comportamiento del número de ejemplares de ganadería del predio Laguna Negra, localizado en el sector El Ocho Letras. Se definieron como variables el número de ejemplares de ganado en las distintas edades del ciclo de vida. Los ejemplares pasan por las distintas edades, en cada una de ellas demoran un año. Para la definición de los flujos entre una y otra edad se consideraron flujos retardados. Se considera una estrategia de control para limitar el número de ejemplares mayores de un año. Se planteó un modelo matemático en tiempo discreto y se realizó simulación, obteniéndose un comportamiento oscilatorio periódico para las variables. Algunas de las principales contribuciones del trabajo están relacionadas con: análisis del efecto de variables exógenas sobre el comportamiento de las variables de nivel; planteamiento del control del número de ejemplares mayores de un año de edad basado en venta de crías hembra; desarrollo del modelo matemático a tiempo discreto, utilizando el método de cadena con flujos retardados.

PALABRAS CLAVE: Dinámica de sistemas, modelado, flujos retardados, ganado.

MODELO MATEMÁTICO DA DEMOGRAFIA DO GADO DE UMA PROPRIEDADE DO SETOR EL OCHO LETRAS

RESUMO

Na pecuária El Ocho Letras o gado é uma importante causa de degradação da vegetação, zonas húmidas naturais e recursos hídricos. Isso leva à necessidade de conhecer e compreender a evolução do número de cópias dos diferentes idades de gado, ao longo do tempo, e o efeito de diversas variáveis exógenas sobre o comportamento do sistema. A este respeito, no presente estudo se aplica a Dinâmica de Sistemas para representar o comportamento do número de cabeças de gado fazenda Laguna Negra, localizada no setor El Ocho Letras. Se definiram como variáveis o número de gado em diferentes idades do ciclo de vida. Eles passam por diferentes idades, em cada uma delas demoram um ano. É considerada Para a definição de fluxos entre uma e outra idade se consideraram fluxos retardados. Uma estratégia de controle para limitar o número de animais maiores de um ano. Um modelo matemático em tempo discreto foi levantada e simulação foi realizada para dar um comportamento oscilatório periódico para variáveis. Algumas das principais contribuições do projeto estão relacionadas a: análise do efeito de variáveis exógenas sobre o comportamento das variáveis de nível; planejamento do controle do número de animais maiores a um ano de idade com base na venda de filhotes femininas; desenvolvimento do modelo matemático de tempo discreto, usando os fluxos método cadeia retardados.

PALAVRAS-CHAVE: modelagem dinâmica de sistemas, fluxos atrasadas, gado.

1. INTRODUCTION

Cattle ranching in Colombia generates significant environmental impacts on soil, water, air, energy, and biodiversity. In fact, cattle ranching has a significant impact on natural ecosystems and water resources. The main mechanism for transforming habitats and ecosystems in Colombia is deforestation, and its main cause is the colonization and expansion of the agricultural frontier. Cattle ranching is the "activity which occupies the greatest area of transformed land in Colombia, affecting natural ecosystems. A large percentage of forests have been lost in Colombia. Cattle ranching is the main activity that has occupied these deforested areas" (Murgueitio, 1999).

In the El Ocho Letras region, there is a large area of land that has been deforested and occupied mainly by cattle ranching. The productive systems in the El Ocho Letras region include agrarian systems, livestock systems, timber extraction activities, and rural tourism. The major productive system in the area is cattle ranching, followed by monoculture potato farming. Cattle ranching is extensive and, in some areas, semi-intensively managed. It is focused on milk production and dual-purpose production (Fundación Pangea, 2007: pp. 29, 30, 41, 42).

In the El Ocho Letras region, cattle ranching generates significant degradation of natural wetlands, which supply water to the Chinchina river basin, whose water mirror supports some animal species (Fundación Pangea, 2007: pp. 29-30). The evolution of the age of cattle over time has an impact on the dynamic of deforestation and the dynamic of natural wetland degradation. It is therefore necessary to know and understand the evolution of the number of cattle of different ages over time. However, there is no knowledge on this issue.

One methodology that allows us to have this knowledge is system dynamics, which has been proved effective to represent and study environmental and agricultural systems by incorporating scientific knowledge. It is not limited to obtaining the mathematical model and analyzing the results of the simulation, but also allows us to understand: i) the causes of the behavior of the system's characteristic variables; ii) the interaction among the exogenous variables and the interaction among the system's characteristic variables; iii) the effect of the exogenous variables on the system variables. The simulation based on a mathematical model allows us to establish the behavior of the system's characteristic variables for different scenarios of the exogenous variables (Sterman, 2000; García, 2003: p. 28). As such, system dynamics is important to understanding and analyzing the dynamic of natural wetland degradation in El Ocho Letras and therefore contributes to proposing alternatives for managing large numbers of cattle so that terrain used by natural wetlands is not occupied.

The study by Schaffernicht (2006: pp. 213-214) shows that there are cases in which cattle go through various stages, such that when they leave one stage, they enter another, and the state variables are these stages. Therefore, a chain structure can be used. One example of this is when there are age groups and the head of cattle move through each of the ages. When they leave one age group, they enter the next. For example, on p. 215, a human resources system is developed using system dynamics. A company whose professionals perform projects is considered. The state variables are: vacancies, new professionals, and experienced professionals. The exogenous variables are: retirement rates, productivity of experts, and productivity of new professionals. A chain behavior is present: after a certain amount of time, new professionals become experienced professionals, and then they retire. The fluxes between one level and the next are delayed fluxes, meaning that each flux is equal to the previous one, but with a time delay. The resulting simulation shows that the evolution of "experienced professionals" has the same form as that of "new professionals," but with a time delay.

The study by Dieguez & Cameroni (2014) analyzed cattle ranching productivity on an extensive cattle farm with variations in feed supply. To propose the model and simulation, the current version of MEGanE software was used to propose a deterministic model. This software uses a model called the Extensive Cattle Exploitation Model, which is a physical and biological "prey-predator" model described by differential equations in continuous time. The characteristic variables or state variables are: grass height (AP for its abbreviation in Spanish) and live weight (PV for its abbreviation in Spanish). The structure of this model is described by a causal diagram which indicates the relationships of cause and effect between the state variables and other variables. The maximum possible quantity of feed (coefClima) associated to climate risk was considered to be a parameter variable. Random values were introduced to this variable in six variability scenarios. It was observed that when the variability amplitude of coefClima was increased, the result was an increase in the variability of the system and a reduction in the average values.

The study by Chajin & Jiménez (2010) developed six systems to represent the behavior of an intensive bovine production system. It used the system dynamics and systems thinking methodologies. The set of six systems was called the "Intensive Bovine Ranching Simulation Model," and each system is denominated a prototype.

The sectors or components of each prototype are: demographic, biophysical, productive, and economic. The set of six systems is prepared in the SIPROBI 1.0 software tool, which, in turn, is supported by the AGRODISI software. Each system uses differential equations in continuous time, and the simulation was performed for 130 months.

The first prototype is called "double demographic purpose," and it allows us to represent the demographics of a double-purpose cattle ranching system (milk and meat production). It shows the behavior of the bovine population. The categories of age, gender (female or male), and reproductive state are used. The level variables or state variables correspond to the number of cattle in each category, the number of births, and the number of discarded animals.

The second prototype is called "demographic and financial intensive feeding" and allows us to represent demographic and financial characteristics of a bovine ranching system with intensive feeding. Females are not considered. Net profits depend on income minus expenses. Income is due to the sale of discarded animals and the sale of fattened steers. Some of the most important state variables are: number of partially fattened steers, number of steers in the fattening process, number of discarded animals, income from the sale of discarded animals, income from the sale of fattened steers, total income, expenses, and net profits.

The third prototype is called "demographic and financial intensive dairy farming" and allows us to represent demographic and financial characteristics of an intensive bovine farming system with milk production. Males are discarded at birth. Income is due to: sale of the milk generated by the cows in their lactation period and the sale of discarded animals. Some of the most important state variables are: number of milk-giving cows (those who become adults and have high-quality genetics and are subjected to fertilization), number of pregnant cows, number of pregnant and lactating cows, income from the sale of male calves, income from the sale of milk, income from the sale of discarded animals, income from the sale of embryos, expenses due to the cost of production activities, and net profits.

The fourth prototype is called "biophysical component" and allows us to represent the system's performance and productivity in relation to biophysical characteristics for an intensive bovine farming system. Live weight and beef purchase requirement depend on variables such as load capacity, type of feed, climate conditions, and soil types. The beef purchase requirement is the difference between load capacity and live weight. The exogenous variables considered are: i) grazing system, whether intensive grazing or confinement; ii) area, which is the land available for implementing a grazing system; iii) types of grass, of which four are considered; iv) feed offered, which is the amount of feed available on the farm; v) rejected feed; vi) feed consumed; vii) feed energy; viii) total energy, which is the energy provided by the feed and by the supplement. Some of the most important state variables are: sale, discarded animals, number of grass types in paddock 1, and number of grasses in paddock 2.

The fifth prototype is called "general model for intensive feeding" and consists of adding together prototypes 2 and 4. It allows us to represent demographic and economic characteristics and productivity in terms of biophysical characteristics of an intensive feeding bovine farming system. Some of the most important state variables are: partially fattened steers, steers in the fattening process, fattened steers, live weight, number of discarded animals, income for the sale of fattened steers, expenses, and net profits. Some of the most important exogenous variables are: type of grass planted, farm pairing, rainy seasons, and drought. Load weight is the difference between the maximum load capacity and live weight, and it is what allows us to define the beef purchase requirement.

The sixth prototype is called "grass growth and weight gain" and allows us to represent bovine weight gain characteristics for an intensive bovine farming system in terms of grass growth and the amount of grass provided. Some of the most important state variables are: amount of grass cut, amount of grass stored in a silo, weight of the animals, and supplement purchased. Some of the most important exogenous variables are: area for grass planting, grass productivity, maximum load weight, and feeding requirement for each animal.

This study applies the system dynamics methodology to describe the evolution of the number of head of cattle on the Lagun Negra farm located in the El Ocho Letras region. We propose a discrete time model on which to perform the simulation.

The study's general goal is to mathematically represent over time the evolution of the number of cattle on the Laguna Negra farm, which is located in the El Ocho Letras region. Specifically, the number of cattle includes the number of cows, number of heifers, number of female calves, and number of male calves. In turn, this objective includes the following specific objectives:

- Qualitatively establish the relationships of influence between the exogenous variables and the endogenous variables, that is, fluxes, levels, and intermediate variables.
- Obtain the evolution curves for the variables to be studied over time.
- Determine the effect of changes in the exogenous variables on the temporal evolution of the level-type variables.

In this document, the information in the section titled "Data summary," the section titled "Problem definition," the subsection titled "Definition of data for the level and flux diagram and for the simulation" and the subsection titled "Endogenous explanation" is taken from previous sections: the subsection titled "Frontier model table," the section titled "Discrete time simulation model formulation," and the section titled "Model validation." If the reader considers these first sections unnecessary, they may be skipped.

2. METHODOLOGY

The system dynamics model used in this study includes the following sections: data summary; problem definition; dynamic hypothesis formulation; model formulation; and model validation.

The methodology's description is shown in **Table 1**. It is proposed based on Carvajal & Arango (2011: pp. 269-270); Carvajal, Arango, Arango & Younes (2011: pp. 61-65); Tedeschi et al. (2011); Sterman (2000: pp. 89-103); Aracil (1995: pp. 21, 36, 39, 41, 42, 58, 59, 65); Schaffernicht (2006: pp. 34-38, 49, 67, 68, 69, 81, 111).

Data summary

General information on Laguna Negra

The area belongs to the buffer zone around the Los Nevados National Natural Park in the municipality of Villa Maria (Caldas). Its elevation is 3,853 meters above sea level, its average environmental temperature is 11°C (Toro et al., 2012: p. 25), and it has a precipitation level of 873mm per year. The climate presents a bimodal regimen in which the rainiest months are April and December, while the months of drought are January and July. The farm measures 296 hectares in total, which are distributed as follows: 207.2 hectares of cattle ranching and potatoes; 88.9 hectares for conservation. There are 30 hectares of land in a rest phase which are used to cultivate potatoes.

The high elevation above sea level implies that the availability of oxygen is very low. This is a significant disadvantage for cattle ranching since the animals must make a great deal of effort to obtain the oxygen required for their cells. The performance regarding productivity of milk and meat is lessened, and all processes are slower.

<table>
<tr><th colspan="2">TABLE 1. METHODOLOGY DESCRIPTION</th></tr>
<tr><th>Phase</th><th>Description</th></tr>
<tr><td>Collection and analysis of information about the phenomenon to be studied (Data summary section)</td><td>The information about the study area and the process or situation to be studied is expressed: i) biophysical parameters of the study area, including climate, soil, topography, and others, ii) data on the production system. This information allows us to understand the context and situation being studied and is also input so that the entire study can be performed in a manner coherent to reality.
To collect the information, a bibliographic consult was performed, but the majority of the information was obtained through visits to the site and interviews made of the following individuals: i) the farm owner, ii) a rural inhabitant of the area, iii) two professors at Universidad de Caldas. The following subsections are included:
– General information on the Laguna Negra farm
– Demographic information
– Beef sales.</td></tr>
<tr><td>Problem definition (Problem definition section)</td><td>Based on the information obtained, basic aspects are defined regarding the phenomenon under study. These are:
– The problem to be addressed, that is, the problem to be studied.
– The variables of greatest interest to the end users of the results of the final model.
– The real problem that is at the root of the problem to be addressed.
– The model’s purpose.
– The time horizon.</td></tr>
<tr><td>Dynamic hypothesis formulation (Dynamic hypothesis formulation section)</td><td>The dynamic hypothesis is a theory of the dynamic system’s structure, and it indicates how the system or situation being studied arises and operates. The following subsections are proposed:
i) Definition of the information for the level and flux diagram and the simulation. Information and values necessary to develop the mathematical model and the respective simulation are defined.
ii) Endogenous explanation in which the model’s structure, the interaction between variables, and the causes of the variables’ behavior are explained. It is divided into the following subsections: cattle ages, delayed fluxes, initial acquisition of cattle, generation of new calves, sale of cattle, and control of the number of cattle older than one year.
iii) Frontier model table: this includes the definition of the exogenous and endogenous variables. In turn, the endogenous variables are divided into level-type variables, flux-type variables, and intermediate variables. This task is input for the correct development of the map of reserves and fluxes and for the correct proposal of the equations that describe the system.
iv) Map of reserves and fluxes: these are diagrams which mainly indicate the relationships between the level-type and flux-type variables which are formed as a consequence of the cause-and-effect relationships and feedbacks. In the map, the level-type variables are represented by rectangles, the flux-type variables are represented by arrows, and the intermediate and exogenous variables do not have geometric shapes.</td></tr>
<tr><td>Simulation model formulation (Simulation model formulation section)</td><td>This is the construction of the mathematical model. In discrete time, the equations that allow us to quantify the level-type variables are defined. In turn, this implies definition of the equations for intermediate variables and flux-type variables, as well as the definition of the initial conditions. It includes the following subsections:
– Definition of fluxes and equations for cattle: the equations that allow us to quantify the variables that correspond to the cattle’s ages are defined, as well as the initial conditions, the intermediate variables, and the fluxes.
– Definition of the number of female calves to be sold: the equations that allow us to quantify the number of female cattle to be sold are defined. This is an intermediate variable that is required for the level-type variables.</td></tr>
</table>

Model validation (Model validation section)	The equations that were formulated in discrete time are taken to a simulation in Matlab in order to generate graphs that mainly indicate the temporal evolution of the level-type variables, but that also indicate the evolution of the flux-type variables or intermediate variables. Based on the simulations and information from reality, the validity of the structure and the system's behavior are verified, and modifications and improvements are made as necessary, although the details of performing these improvements and adjustments are not shown in this document. In the simulation: i) Several scenarios are considered: first, that which corresponds to real situations, then that which corresponds to extreme conditions of the exogenous variables. ii) A numerical simulation is performed by implementing the model in Matlab software. Simulations are performed using extreme values of the exogenous variables. iii) The coherence of the simulated behavior with reality is analyzed, as is transient behavior. iv) The influence of the exogenous variables on the cattle populations is analyzed. The following subsections are included: – Basic numerical simulation. – Behavior under extreme input conditions.

The nutritional quality of the grass is low in the Laguna Negra area due to these conditions. In addition, the low level of precipitation in the area causes grass production to be low. Rotational grazing is practiced, which consists of alternating the use of paddocks. This is performed every 20 days.

Demographic information

The cattle on the farm are all Norman cattle and double-purpose: for the production of milk and high-quality meat. This breed is characterized by: i) its capacity to move over expansive and rugged terrain in search of food; and ii) its high resistance to illnesses, such that deaths are negligible.

On the land, there are approximately 200 cows, which implies a ratio of approximately one head of cattle per hectare. The productive life of the cattle begins at the age of approximately 3 years. The frequency of births undergone by each head of cattle is approximately one calf per year. There is normally a lapse of 365 days between each birth. For the percentage of females among the births, a value of 50% may be used. The gestation period for a cow is 9 months. The lactation period is approximately 300 days, and calves are weaned at 7 months. The cow is then dried 60 days before the subsequent birth, that is, milking is stopped. A female calf takes 24 months to become an adult cow. At 24 months, a bull is provided so that the cow may begin to generate calves and, therefore, to produce milk. It is not necessary to buy calves since the cows generate their heifers, for which impregnation is necessary. Generally, when a cow reaches between 5 and 7 births, it is sold for meat. As such, the cow is sold at approximately 8 years of age.

Sale of cattle for beef

There is a limit to the number of cattle that can occupy a territory which is given by the area available and corresponds to approximately 1 cow per hectare. This limit includes male and female cattle older than one year. When the number of cattle older than one year reaches said limit, in order to keep from exceeding it, no more cattle are introduced and, in addition, cattle must be sold in accordance with the following options: i) sale of heifers and male calves 1 week old; and ii) sale of discarded cows. Discarded cows are those animals which, due to their age, have a low level of milk production and are therefore sold for meat. For the sale of male cattle, there are two options: i) sell them at one week old; or ii) wean them at 7-8 months, fatten them, and sell them at 3 years of age.

Problem definition

The problem to be studied and represented is cattle ranching, including the generation of cattle for sale. The variables to be studied are: overall number of cattle, number of productive cows, number of male cattle, and number of heifers. Cattle ranching yields significant economic earnings, which is a motivation to use land for ranching. This implies the use of areas initially occupied by plant life, including natural wetland areas.

The model's purpose is to describe the evolution over time of the number of cattle on the Laguna Negra farm, which is located in the El Ocho Letras region, including number of cows, number of heifers, number of female calves, and number of male calves. The effect of changes in the exogenous variables on the evolution over time of the level-type variables is considered.

For the time horizon, the following factors or information are considered:

- Bozic (2009: p. 50) shows in a graph the evolution of the number of milk cows by taking a period from 1975 to 2005, that is, 30 years. However, appreciable changes can be seen over periods of 20 years.
- Bozic (2009: p. 16) shows in a graph the evolution of the number of milk cows in the USA during the period of 1950 to 2005, that is, 55 years. However, appreciable changes can be seen over periods of 20 years.

Therefore, a time horizon of 25 years is used.

Dynamic hypothesis formulation

This section includes: i) definition of the information necessary for the level and flux diagram and for the simulation; ii) endogenous explanation; iii) the model's frontier table: endogenous variables, exogenous variables; iv) map of reserves and fluxes; v) discussion and conclusions.

Definition of information for the level and flux diagram and for the simulation

The following information is taken to facilitate the representation of the level and flux diagram and for the simulation: i) age at which the female cattle have their first calf: at 3 years; ii) percentage of females among births: 50%; iii) limit number of cattle older than one year: 200; iv) age of cows at sale for meat: at 8 years, they are sold without having their sixth calf; v) age of sale for male cattle: at one week old; vi) age of sale for female cattle: at one week; vii) simulation time: 60 years.

Endogenous explanation

The most important cattle variables are: i) heifers, that is, female cattle between 0 and 3 years old; ii) productive cows producing milk between 3 and 8 years old; iii) number of male calves; iv) number of female calves.

Cattle ages. Different cattle ages are established instead of having only one since each of these ages generates different effects. The female cattle were divided into various ages, which range from type 1 heifers to type 5 cows. Three ages are considered for heifers, and five ages are considered for cows. Each of these ages lasts approximately one year. As such, when an animal becomes a heifer or a cow, a period of one year elapses for the age group being entered. At the end of that year, the animal moves into the following age group. Newly acquired animals enter the type 1 heifers stage, in which they remain for one year. They then move to type 2 heifers, then to type 3 heifers, then to type 1 cows, then to type 2 cows, type 3 cows, type 4 cows, and to type 5 cows, and are then sold for meat. They spend one year in each of these age groups. The difference between cows and heifers is that cows have calves and produce milk. Animals of type 1 cow through type 5 cow are those that produce milk, which generates the main economic gains. The variable "number of cows" (N_{vc}) refers to the number of cows that produce milk and is defined as the sum of the number of cows from type 1 cows (N_{vc1}) through type 5 cows (N_{vc5}). As such, the number of heifers and calves are not considered. The variable "number of animals

1" (N_{eje1}), refers to the number of animals above one year old and is defined as the sum of "number of type 2 heifers" (N_{tnr2}), "number of type 3 heifers" (N_{tnr3}), "number of type 1 cows (N_{vc1}), ..., number of type 5 cows" (N_{vc5}). Therefore, "number of type 1 heifers" (N_{tnr1}), is not considered since this corresponds to the age range of zero to one years.

Delayed fluxes. The fluxes between different ages are delayed fluxes: flux f_{ob} is equal to f_o delayed one year; flux f_{oc} is equal to f_{ob} delayed one year; flux f_1 is equal to f_{oc} delayed one year; flux f_{1b} is equal to f_{oc} delayed one year; flux f_{1c} is equal to f_{1b} delayed one year; flux f_{1d} is equal to f_{1c} delayed one year; flux f_{1e} is equal to f_{1d} delayed one year. A similar delayed flux example or phenomenon can be found in the text by Schaffernicht (2006: pp. 217 and 218).

Initial acquisition of animals. In principle, at the initial time (t_o), heifers appear because of purchase. Only type 1 heifers are bought, and purchase is only made at the initial time. After the initial time, type 1 heifers do not appear due to purchase, but rather from pregnant cows that belong to the farm itself. In turn, these cows have appeared as a consequence of the initially purchased heifers. The initial purchase of heifers is what detonates or initiates the temporal evolution of the variables.

Generation of new calves. The generation of calves occurs approximately every year. Of said calves, there is a percentage of females and a percentage of males. The male calves are sold. Of the female calves, a portion is sold in order to limit the number of animals older than one year. The female calves that are not sold become type 1 heifers. When a new animal becomes a type 1 cow, it generates a new calf, which may be male or female. Likewise, when a new animal becomes a "type 2 cow," a "type 3 cow," a "type 4 cow," or a "type 5 cow," it generates a calf that may be male or female. If the calf is male, then it is sold at one week old. But if the calf is female, it can enter "type 1 heifer" or be sold if this is required in order to limit the number of animals older than one year.

Sale of cattle. The female cattle, after being type 5 cows, reach 8 years of age and are sold for meat. The male cattle are sold when they reach one week old. The number of cattle older than one year cannot exceed the limit value which is given by the amount of available area. The sale of cows at 8 years old and the sale of male calves at one week old contribute to this end. However, it is not sufficient, and therefore a portion of the female calves are sold at a week old. The number of female calves sold is decided such that the number of cattle older than one year does not exceed the limit value.

Control of the number of cattle older than one year. The definition of this control is a complex task that requires analyzing various options. As a result, the following option was selected: selling female calves. This option allows for: i) maintaining a high number of type 1 through type 5 cows, thereby maintaining high milk production; ii) obtaining whole numbers in the development of the Matlab simulation algorithm. In addition, the following characteristic is present: the sale of female calves must be done with foresight, that is, by defining the number of female calves that must be sold at moment k so that at moment k+1, there is not an excess of cattle older than one year.

Frontier model tables

This includes endogenous and exogenous variables. The endogenous variables are classified as flux-type variables, level-type variables, and intermediate variables.

Level-type variables:

- Number of type 1 heifers: number of female cattle between zero and one year old that are present on the farm (N_{tnr1}).
- Number of type 2 heifers: number of female cattle of between one and two years old present on the farm (N_{tnr2}).
- Number of type 3 heifers: number of female cattle of between two and three years old present on the farm (N_{tnr3}).

- Number of type 1 cows: number of cows present on the farm that have had only their first calf and are between three and four years old (N_{vc1}).
- Number of type 2 cows: number of cows present on the farm that have had their second calf and are between four and five years old (N_{vc2}).
- Number of type 3 cows: number of cows present on the farm that have had their third calf and are between five and six years old (N_{vc3}).
- Number of type 4 cows: number of cows present on the farm that have had their fourth calf and are between six and seven years old (N_{vc4}).
- Number of type 5 cows: number of cows present on the farm that have had their fifth calf and are between seven and eightyears old (N_{vc5}).

Flux-type variables:

- f_o is the input speed of type 1 heifers on the farm;
- f_a is the input speed of type 1 heifers on the farm acquired by purchase;
- f_{ob} is the speed of appearance of type 2 heifers on the farm;
- f_{oc} is the speed of appearance of type 3 heifers on the farm;
- f_1 is the speed of appearance of type 1 cows on the farm;
- f_{1b} is the speed of appearance of type 2 cows on the farm;
- f_{1c} is the speed of appearance of type 3 cows on the farm;
- f_{1d} is the speed of appearance of type 4 cows on the farm;
- f_{1e} is the speed of appearance of type 5 cows on the farm;
- f_{1f} is the speed of disappearance of type 5 cows on the farm due to their sale for meat.

Intermediate variables:

- Number of male calves: this is the number of newborn male calves.
- Number of female calves.
- Number of productive cows (N_{vc}): this is the number of active cows, which includes "number of type 1 cows" through "number of type 5 cows".
- Number of type 1 cattle (N_{eje1}): number of cattle older than one year
- Number of female calves for sale ($N_{chbvent}$): this is the number of female calves to be sold.
- Number of future test (N_{pbft}): this is the number of cattle older than one year for a future moment k+1, but calculated at moment k under the supposition that there is n sale of female calves.

Exogenous variables:

- Number of heifers purchased at the initial time t_o: this is the purchase of type 1 heifers at the initial time (N_{tnrto}). It is related to the cattle ranching surface at initial time $t=t_o$.
- Percentage of female births among cows.
- Limit number of cattle older than one year.

Map of reserves and fluxes

Figure 1 shows the map of reserves and fluxes. This map includes the level-type variables in rectangles; the flux-type variables in arrows; and, for the intermediate variables, no geometric shape is used (v. Carvajal, 2011: p., 270; Sterman, 2000). The sale of female calves implies a balance feedback loop such that the number of cattle does not grow infinitely. See the discussion in Tedeschi, Nicholson & Rich (2011: p. 105).

Formulation of the discrete time simulation model

The number of heifers, productive cows, and males behaves in bursts. It is therefore simpler to perform a discrete time model than a continuous time model. The presence of delayed fluxes makes continuous time modeling complex (Aracil, 1999: p. 12). In contrast, it is highly simple to obtain the delayed value of any variable in discrete time, and the method is to use indices to design and save those values. Therefore, a discrete time model was performed using Matlab.

Definition of fluxes and equations for cattle

Since discrete time is being considered, a counting index denominated k is used, and the initial time is defined as k=1. SThe following must be considered: i) initially, type 1 heifers are purchased by their owner, assuming a value of 50 type 1 heifers; ii) initially, cows for obtaining milk are not acquired since, instead, type 1 heifers are acquired which will later grow into cows and produce milk. Therefore, the number of type 2 heifers, type 3 heifers, type 1 cows,... type 5 cows is zero at the initial time. In addition, at initial time k=1, only the input flux of type 1 heifers is not zero; the remaining fluxes are zero. The purchase of heifers flux is a burst which is defined as:

$$f_\alpha\big|_{k=1}=50\ ,\ f_\alpha\big|_{k>1}=0$$

The fluxes for heifers and cows are delayed fluxes, as was mentioned in the "Endogenous explanation" section, "delayed fluxes." For this, we partly relied on the text by Schaffernicht (2006: pp. 217- 218). To calculate the input flux of type 1 heifers, represented by f_o, we need: i) number of female calves sold in order to keep the number of cattle older than one year from exceeding the limit value: $N_{chbvent}\big|_k$; ii) the percentage of female calves among the cows' births: N_{ncmhb}; iii)number of calves, including males and females: $\left(f_1\big|_k+f_{1b}\big|_k+f_{1c}\big|_k+f_{1d}\big|_k+f_{1e}\big|_k\right)$; iv) input of type 1 heifers by purchase: $f_{a.}$ Flux f_a only has a value other than zero at the initial time. Flux f_0 fulfills the following equation:

$$f_0\big|_k=f_\alpha\big|_k+N_{ncmhb}\left(f_1\big|_k+f_{1b}\big|_k+f_{1c}\big|_k+f_{1d}\big|_k+f_{1e}\big|_k\right)-N_{chbvent}\big|_k$$

The number of cattle in each of the different age groups is equal to the input flux at the corresponding age:

$$N_{tnr1}\big|_k=f_0\big|_k\ ,N_{tnr1}\big|_{k=1}=f_\alpha\big|_k\qquad N_{tnr2}\big|_k=f_{ob}\big|_k\ ,\ N_{tnr2}\big|_{k=1}=0;$$

$$N_{tnr3}\big|_k=f_{oc}\big|_k\ ,$$

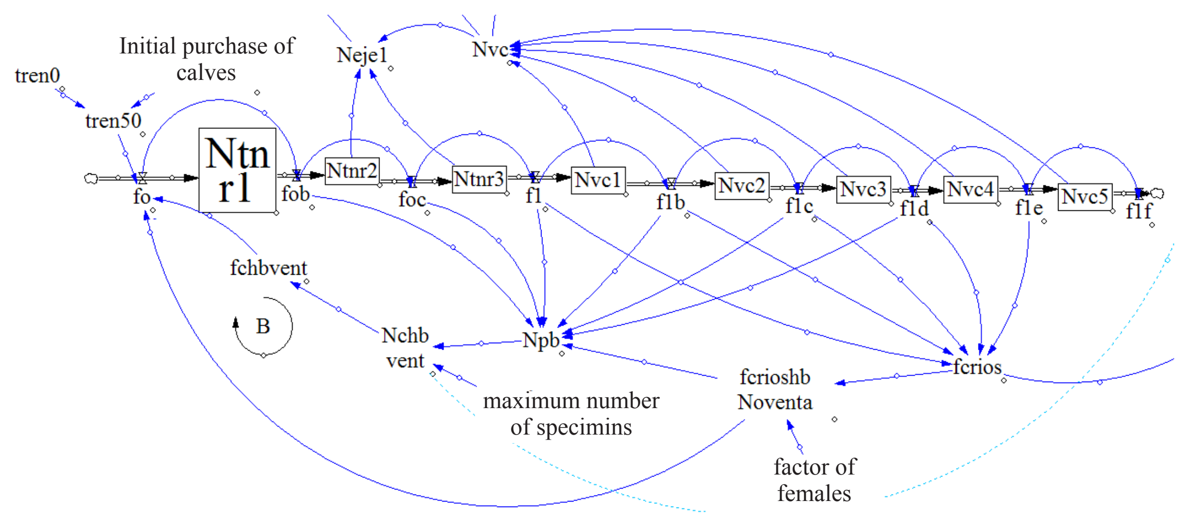
$$N_{tnr3}\big|_{k=1}=0\qquad N_{vc1}\big|_k=f_1\big|_k\ ,\ N_{vc1}\big|_{k=1}=0\qquad N_{vc2}\big|_k=f_{1b}\big|_k\ ,$$

$$N_{vc2}\big|_{k=1}=0$$

$$N_{vc3}\big|_k=f_{1c}\big|_k\ ,\ N_{vc3}\big|_{k=1}=0\qquad N_{vc4}\big|_k=f_{1d}\big|_k\ ,\ N_{vc4}\big|_{k=1}=0$$

$$N_{vc5}\big|_k=f_{1e}\big|_k\ ,\ N_{vc5}\big|_{k=1}=0$$

Figure 1. Level and flux diagram.
Source: created by authors using Vensim software.



Definition of the number of female calves to be sold

Female calves are sold so that the number of cattle older than one year does not exceed the limit value.

At any given moment, the cattle older than one year include from type 2 heifers, which are heifers of between 1 and 2 years old, through type 5 cows. Type 2 heifers, before becoming type 2, a year before had been in the type 1 stage and had also passed through the female calf stage. Therefore, if at a given moment k, a portion of the female calves are sold, the value of the number of type 2 heifers would be reduced at a future moment k+1, and, therefore, so would the number of cattle older than 1 year.

The option of selling a portion of the female calves was chosen to directly reduce the excess of female calves and maintain a high number of productive cows N_{vc}, avoiding excessive values for the number of type 2 heifers and type 3 heifers. This option is also coherent with reality.

A test index N_{pbft}, is considered. It indicates, at moment k, the number of cattle older than one year for a future moment k+1 under the condition that no female calves are sold.

$$N_{pbft}\Big|_k = N_{tnr2}\Big|_{k+1} + N_{tnr3}\Big|_{k+1} + N_{vc1}\Big|_{k+1} + N_{vc2}\Big|_{k+1} + N_{vc3}\Big|_{k+1} + N_{vc4}\Big|_{k+1} + N_{vc5}\Big|_{k+1}$$

$$= f_{ob}\Big|_{k+1} + f_{oc}\Big|_{k+1} + f_1\Big|_{k+1} + f_{1b}\Big|_{k+1} + f_{1c}\Big|_{k+1} + f_{1d}\Big|_{k+1} + f_{1e}\Big|_{k+1}$$

$$= f_o\Big|_k + f_{ob}\Big|_k + f_{oc}\Big|_k + f_1\Big|_k + f_{1b}\Big|_k + f_{1c}\Big|_k + f_{1d}\Big|_k$$

Above, the input flux of type 1 heifers was defined as $f_o|_k$, which includes the term $N_{chbvent}|_k$, which corresponds to the sale of female calves. When there is no sale of female calves, that is, $N_{chbvent}|_k=0$, the input of type 1 heifers is:

$$f_o\Big|_k = f_\alpha\Big|_k + N_{ncmhb}\left(f_1\Big|_k + f_{1b}\Big|_k + f_{1c}\Big|_k + f_{1d}\Big|_k + f_{1e}\Big|_k\right)$$

Replacing this expression of $f_o|_k$, in the expression $N_{pbft}|_k$, we obtain:

$$N_{pbft} = f_\alpha\Big|_k + N_{ncmhb}\left(f_1\Big|_k + f_{1b}\Big|_k + f_{1c}\Big|_k + f_{1d}\Big|_k + f_{1e}\Big|_k\right) + f_{ob}\Big|_k + f_{oc}\Big|_k + f_1\Big|_k + f_{1b}\Big|_k + f_{1c}\Big|_k + f_{1d}\Big|_k$$

The above equation allows us to calculate the value of N_{pbft} in the simulation. The surplus is defined, that is, the excess of cattle older than 1 year, which would occur if there is no sale of female calves:

$$N_{pbft} - N_{\lim}$$

This surplus is only calculated for when N_{pbft} is greater than the limit number N_{lim}. Two cases exist: i) N_{pbft} is greater than the limit number N_{lim}, in this case, female calves must be sold, such that the number of female calves to be sold is greater than zero, $N_{chbvent}|_k>0$; ii) N_{pbft} is lesser than or equal to the limit number N_{lim}, in this case, no female calves must be sold, such that the number of female calves to be sold is zero, $N_{chbvent}|_k=0$.

The number of female calves to be sold equals the surplus:

$$N_{chbvent}\Big|_k = N_{pbft} - N_{\lim}\text{ , if } N_{pbft} > N_{\lim}$$

As such, the algorithm logic is

$$N_{pbft} = f_\alpha\Big|_k + N_{ncmhb}\left(f_1\Big|_k + f_{1b}\Big|_k + f_{1c}\Big|_k + f_{1d}\Big|_k + f_{1e}\Big|_k\right) + f_{ob}\Big|_k + f_{oc}\Big|_k + f_1\Big|_k + f_{1b}\Big|_k + f_{1c}\Big|_k + f_{1d}\Big|_k$$

Si $N_{pbft} \leq N_{\lim}$

then $N_{chbvent}\Big|_k = 0$

unless

then $N_{chbvent}\Big|_k = N_{pbft} - N_{\lim}$

end

$$f_o\Big|_k = f_\alpha\Big|_k + N_{ncmhb}\left(f_1\Big|_k + f_{1b}\Big|_k + f_{1c}\Big|_k + f_{1d}\Big|_k + f_{1e}\Big|_k\right) - N_{chbvent}\Big|_k$$

Validation of the model

In this section, the equations formulated in discrete time are taken to the Matlab software simulation. This section includes: i) basic numerical simulation; ii) behavior under extreme input conditions; iii) discussion and conclusions.

Basic numerical simulation

Matlab software is used. **Figure 2** shows how index k corresponds with time for the case in which discrete time modeling is used.

Figure 2. Ratio between index k and time for the case using discrete time modeling.

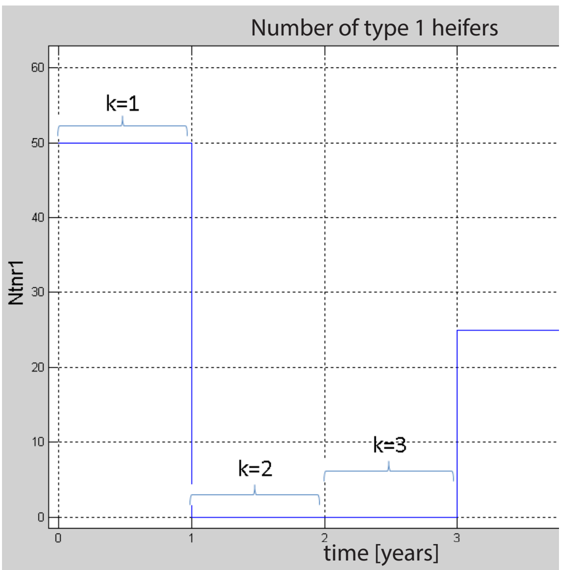


Figure 3 presents the evolution of the number of type 1 heifers (N_{tnr1}) and the number of type 2 heifers (N_{tnr2}). A first characteristic that can be observed is that the value of N_{tnr1} begins at 50. This is due to the fact that this is the number of cattle purchased at the initial time. A second characteristic is that N_{tnr1} becomes zero and later becomes greater than zero at moment 3 years. This is due to the fact that generation of new calves occurs at moment 3 years, which is also due to the fact that the cattle purchased must pass through stage type 1 heifer, then type 2 heifer, then type 3 heifer to later generate new calves to become type 1 cows. This is the first time that new female calves are generated by the farm's own cattle. A third characteristic is that N_{tnr1} takes on a periodical oscillatory behavior which moves from zero to 50. This is due to the sale of female calves, considering that if no female calves were sold, an indefinite increase would occur. A fourth characteristic is that the period of oscillation is seven years. This means that if a random point is taken in the evolution of N_{tnr1}, the same point will be repeated seven years before or after.

It can be observed that the value of N_{tnr2} is N_{tnr1} delayed one year, due to the fact that the fluxes for N_{tnr2} are the fluxes of N_{tnr1} but delayed. This is coherent with reality. Just as the number of type 1 heifers (N_{tnr1}), the variables N_{tnr2}, N_{tnr3}, N_{vc1}... N_{vc5} also undergo an oscillatory behavior which goes from zero to 50 over a period of seven years.

Figure 3. Evolution of the number of type 1 heifers and of type 2 heifers.
Original graph created using Matlab software.

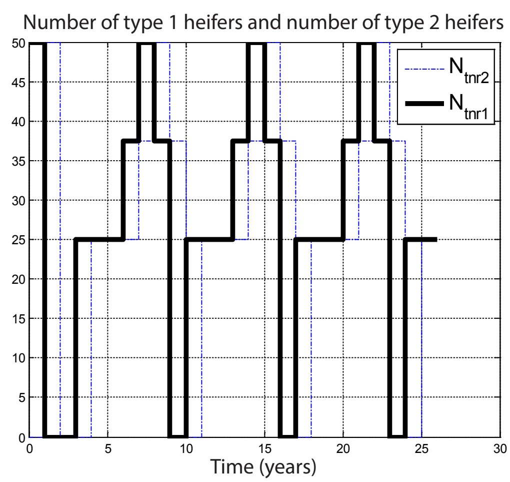


Figure 4 shows the evolution of the number of type 1 heifers through type 1 cows. It can be observed that N_{vc1}, N_{tnr3}, N_{tnr2}, have the same values as N_{tnr1}, but delayed, with different time delays. For example, N_{vc1} is equal to N_{tnr1} but delayed 3 years. This shows that the results are coherent with the idea that was proposed regarding the evolution of the variables. It is also coherent with reality.

Figure 5 shows the evolution of the number of type 1 cows (N_{vc1}), through type 5 cows (N_{vc5}). It can also be observed in this case that variables N_{vc1} through N_{vc5} have the same values, but with a time delay.

Figure 4. Evolution of the number of type 1 heifers through the number of type 1 cows. Original graph created using Matlab software.

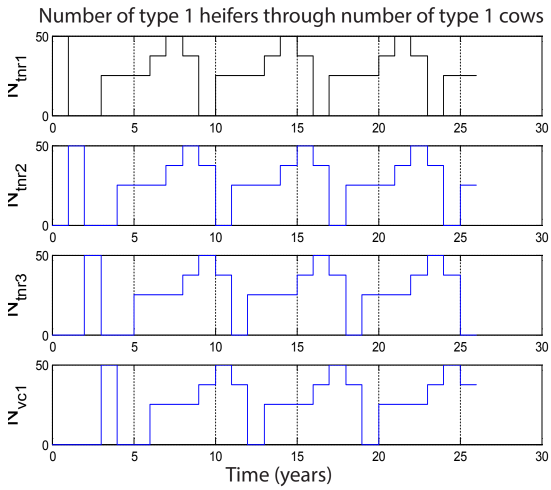


Figure 5. Evolution of the number of type 1 cows (N_{vc1}), through number of type 5 (N_{vc5}).
Original graph created using Matlab software.

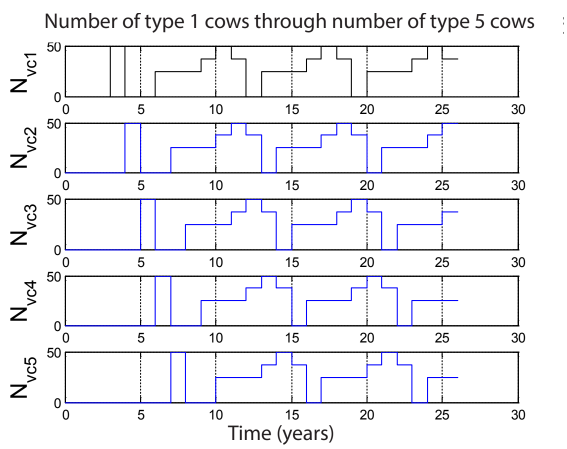


Figure 6 shows the evolution of the number of cattle older than 1 year (N_{eje1}), and of the number of productive cows (N_{vc}). It can be observed that N_{eje1}, begins at zero and slowly increases until, at moment t = 9 years, it reaches the limit value N_{lim}=200 and remains constant at this value. This is coherent with the control mechanism used because when there is an excess of female calves, the mechanism eliminates the female calves such that N_{eje1} will be equal to the limit number N_{lim} without exceeding it.

The number of productive cows (N_{vc}) begins at zero and increases until, at nine years, it converges with a periodic attractor, that is, a repeating oscillatory behavior. A first characteristic observed is that N_{vc} never exceeds N_{eje1}, which is due to the fact that N_{eje1} includes N_{vc} but also N_{tnr2} y N_{tnr3}. A second characteristic is that the value of N_{vc} becomes greater than zero after the three-year mark, which is due to the fact that the purchased cattle must pass through the type 1 heifer stage, then through type 2 heifer, and type 3 heifer before becoming type 1 cows. This transition implies a delay of three years so that the purchased cattle can become type 1 cows. These aspects are coherent with reality.

Figure 6. Evolution of the number of cattle older than one year (N_{eje1}) and number of productive cows (N_{vc}).
Original graph created using Matlab software.

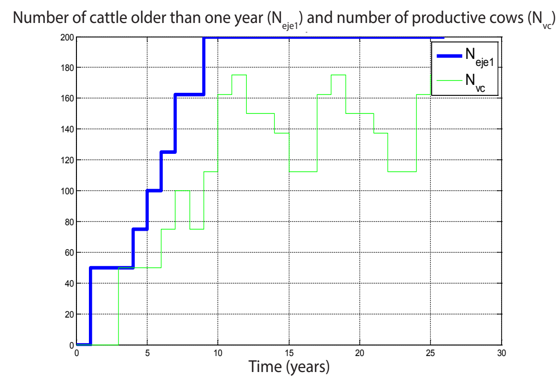


Figure 7. Evolution of the number of female calves sold.
Original graph created using Matlab software.

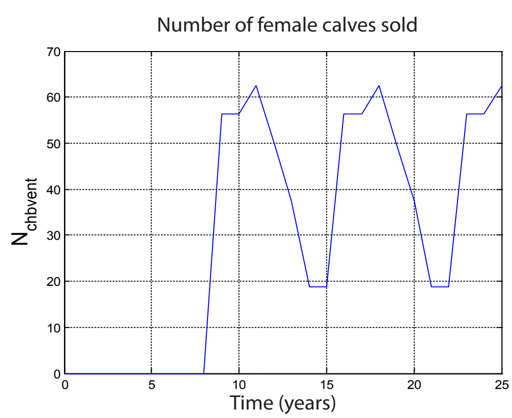


Figure 7 shows the evolution of the number of female calves sold. Its value increases from zero after the nine-year mark and remains greater than zero. This is related to the high production of female calves, which implies that they must be constantly sold.

Behavior with extreme conditions of inputs

To evaluate the quality of the proposed equations in discrete time and the causal structure, we can consider extreme situations for the model's external variables (v. Aracil, 1995: p. 64): for each variable, an extreme low, a nominal value, and an extreme high are taken. See **Table 2**.

TABLE 2. RESPONSE TO VARIATIONS IN THE MODEL'S INTERNAL PARAMETERS. A DISCRETE TIME MODEL IS USED IN MATLAB. NOMINAL VALUES OF THE VARIABLES: NUMBER OF HEIFERS PURCHASED AT INITIAL TIME: N_{tnrto} =50; PERCENTAGES OF FEMALE CALVES AMONG COW BIRTHS: N_{ncmhb}=50 %; MAXIMUM NUMBER OF CATTLE OLDER THAN ONE YEAR: N_{lim}=200.

Parameter undergoing variation	*Value of parameter undergoing variation*	*Number of productive cows (N_{vc})*	*Number of type 1 heifers (N_{tnr1})*
Heifers purchased at initial time (N_{tnrto})	20	123 to 165	15 to 41
	50	112 to 175	0 to 50
	90	110 to 200	0 to 90
Percentage of female calves among cow births (N_{ncmhb})	20%	50	10
	50%	112 to 175	0 to 50
	80%	120 to 200	0 to 50
Maximum number of cattle older than one year (N_{lim})	90	40 to 90	0 to 45
	200	112 to 175	0 to 50
	300	175 to 250	25 to 68

The following is concluded:

- The three parameters that undergo variation have a significant effect on the number of productive cows (N_{vc}), and the number of type 1 heifers (N_{tnr1}).
- An increase in the number of heifers purchased at the initial time (N_{tnrto}) creates an increase in the radius of oscillation N_{vc} and N_{tnr1}. This implies that N_{tnrto} does not have a clear effect of net increase or net decrease on N_{vc} and N_{tnr1}.
- The percentage of female calves among cow births (N_{ncmhb}) has a significant effect on N_{vc} and N_{tnr1}. An increase in N_{ncmhb} generates an increase in the lower limit of the oscillation of N_{vc} and $N_{tnr1,}$ which implies that N_{ncmhb} generates an effective increase of N_{vc} and N_{tnr1}.
- An increase in N_{lim} generates an increase in the lower limit of the oscillation of N_{vc} and N_{tnr1} , which implies that the increase in N_{lim} effectively generates and increase in N_{vc} and $N_{tnr1.}$

3. DISCUSSION AND CONCLUSIONS

The diagrams presented in the text by Schaffernicht (2006: pp. 217 and 218) were the bases for proposing the level and flux diagram and the corresponding mathematical model using Matlab software. The proposal of the map of reserves and fluxes involved a high number of re-proposals and improvements, which is a recognized crucial part of the methodology in system dynamics.

Several tasks were performed based on the level and flux diagram: i) the proposal of the endogenous explanation; ii) the definition of endogenous and exogenous variables.

When the information supplied by the two Universidad de Caldas experts was obtained, we had to reconfigure and arrange the diagrams, mainly those parts related to how to control the number of cattle older than one year so that it did not exceed the limit number.

To propose the equations in discrete time, no law of physics or biology or chemistry was used. In its place, the logic of counting the number of cattle in each of the age groups was used for the following situation: in the beginning, there are no cattle in any of the age groups; then some cattle are acquired which enter the first of the age groups; then, after one year, these cattle move into a second age group; then, after one year, they move into a third age group; and so on. This logic or this situation was the principle for proposing the model. The levels are the number of cattle in each of the age groups, and the fluxes are the number of cattle that move from one level to another. This logic or situation implies that: i) the number of cattle in an age group is equal to the number of cattle in the previous age group, delayed one year, considering that there or no losses or removals; ii) any of the fluxes, except the first flux, is equal to the preceding flux, delayed one year. Therefore, the fluxes are delayed. The example in the text by Schaffernicht (2006: pp. 217 and 218) was the basis for this proposal. The count mentioned was performed based on the fluxes since the fluxes are defined based on the time delay mentioned, and then, based on the fluxes, the levels are defined. Then the number of female calf births and male calf births are defined, and, finally, the control action of selling female and male calves is proposed in order to keep the maximum number of cattle older than one year from exceeding the limit number.

This logic was proposed in the form of an algorithm in order to later write it in Matlab. An index was used and denominated k. The results of the simulation were coherent with reality, and no negative values were obtained for the variables such as the number of type 1 heifers, the number of type 2 heifers, the number of cows, and the number of cattle older than one year, among others.

In all of the work for the present study, we can observe that the concepts of automatic control system design are applied, for example, the definition of a condition of balance or reinforcement for the feedback loop. In addition, we can observe that the process is circular and iterative, as indicated by Sterman. In fact, the modifications made to the map of reserves and fluxes implied making modifications to the rest of the tasks, mainly to the endogenous explanation, in the frontier model table and the mathematical model.

The figure of the evolution in the number of type 1 heifers (N_{tnr1}) and type 2 heifers (N_{tnr2}) allows us to conclude the following:

- The value of N_{tnr1} begins at 50. This is due to the fact that this is the number of cattle purchased at the initial time.
- N_{tnr1} becomes zero and then becomes greater than zero at the moment 3 years. This is due to the fact that the purchased cattle must move through the type 1 heifer stage, then type 2 heifers, then type 3 heifers before later reproducing calves and becoming type 1 cows. This is the first time that female calves are generated by the farm's own cows. The generation of calves occurs at the moment 3 years.
- N_{tnr1} takes on a periodic oscillatory behavior which moves from zero to 50. This is due to the sale of female calves since, on the contrary, indefinite growth would occur.
- The oscillation period is seven years. This means that if a random point in the evolution of N_{tnr1} is take, the same point will be repeated seven years before or after..
- The value of N_{tnr2} is equal to N_{tnr1} delayed one year, which is due to the fact that the fluxes of N_{tnr2} are the fluxes of N_{tnr1} but delayed.

The figure of the evolution of the number of cattle older than one year (N_{eje1}) and the number of productive cows (N_{vc}) allows us to conclude that:

- The number of productive cows (N_{vc}) never exceeds N_{eje1}, which is due to the fact that N_{eje1} includes N_{vc} but also N_{tnr2} and N_{tnr3}.
- The value of N_{vc} becomes greater than zero after the three-year mark, which is due to the fact that the purchased cattle must move through the stage of type 1 heifers, then type 2 heifers, then type 3 heifers before becoming type 1 cows. This transition implies a delay of three years for the purchased cattle to become type 1 cows.
- The number of cattle older than one year (N_{eje1}) begins at zero and increases until the nine-year

mark, when it reaches the limit (N_{lim}) and stays at this value. This is coherent with the control mechanism used because when there is an excess of female calves, the mechanism eliminates the female calves so that N_{eje1} is equal to the limit number N_{lim} and does not exceed it.

The table of responses to variations in the model's internal parameters, **Table 2**, was obtained based on simulations. We conclude that:

- The number of heifers purchased at the initial time (N_{tnrto}) has a significant effect on the number of productive cows (N_{vc}), the number of type 1 heifers (N_{tnr1}) and the income from milk (I_{leche}). In increase in N_{tnrto} generates an increase in the radius of oscillation of N_{vc} and N_{tnr1} so that the lower limit of the oscillation decreases and the upper limit increases. Therefore, there is no net increase or net decrease effect on N_{vc} and N_{tnr1}.
- The percentage of female calves among cow births (N_{ncmhb}) has a significant effect on N_{vc}, N_{tnr1} and I_{leche}. An increase in N_{ncmhb} generates an increase in the lower limit of the oscillation of N_{vc} and N_{tnr1}, which implies that there is an effective increase of N_{vc} and N_{tnr1}.
- The maximum number of cattle older than one year, N_{lim}, also has a significant effect on N_{vc} and N_{tnr1}. An increase in N_{lim} generates an increase in the low limit of the oscillation of N_{vc} and N_{tnr1}, which implies that the increase in N_{lim} effectively generates an increase of N_{vc} and N_{tnr1}.

In **Table 2**, the response to variations in the number of heifers purchased at the initial time was obtained based on simulations. We can conclude the following:

- For values of N_{tnrto} between 1 and 90, an increase in N_{tnrto} generates a decrease in the stabilization time for N_{tnr1}, N_{vc}, and N_{eje1}.
- For $N_{tnrto}=50$ and $N_{tnrto}=90$, the stabilization time for N_{tnr1} is zero, which means that N_{tnr1} does not converge toward oscillatory behavior, but rather begins with it.

Some differences between this study and that by Chajin & Jiménez (2010):

- The mathematical model was proposed with discrete time so that a cycle *for* and the index *k* were used for the simulation.
- A chain method with delayed fluxes was used to define the flux-type variables and the level-type variables.
- The control for the number of cattle older than one year was performed based on selling female calves and calculating the test index $N_{pbft}|_k$, which is defined as the value which, for a moment *k*, is the number of cattle older than one year that would exist at a future moment *k*+1 if no female calves are sold.

This study makes the following contributions:

- Development of the map of levels and fluxes for the cattle ranching system at Laguna Negra.
- Development of a discrete time mathematical model for the specific case of cattle ranching at Laguna Negra using the delayed flux chain method to define the flux-type and level-type variables.
- Proposal of a control for the number of cattle older than one year based on the sale of female calves and the calculation of the text index $N_{pbft}|_k$, which is defined as the value, at moment *k*, of the number of cattle older than one year that would exist at a future moment *k*+1 if no female calves are sold.
- Analysis of the effect of exogenous variables on the level variables through simulation for the specific case of the cattle ranching system at Laguna Negra.

The possibility for future work would consider biological variables such as climate and others regarding cattle behavior. This would provide a high level of rigorousness to the study and a high level of correspondence to reality. This would imply rewriting and redoing the entire study, including the following tasks:

- Reviewing the literature to analyze and identify new biological variables.
- Redefining the new biological variables.
- Determining the values of the new biological variables for the case of Laguna Negra.
- Redefining the information for the level and flux diagram.
- Redefining the frontier model tables.
- Creating the level and flux diagram again.
- Reformulating the entire mathematical model to include the new variables.
- Modifying the simulations.

To this end, it would be important to thoroughly analyze the study by Chajin & Jiménez (2010) and to use the biological variables included there.

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